How to express the coordinate transformations

$$\left[\vec{t}\right]_{i-1} = \mathcal{T}_i \circ \left[\vec{t}\right]_i$$

$$= \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix} \left[\vec{t}\right]_i + \vec{l}_i$$

Using Homogenous Coordinates
We can write translation as matrix multiplication

$$
\begin{aligned}
\begin{bmatrix} \vec{t} \\ 1 \end{bmatrix}_{i-1} &= \mathcal{T}_i \circ \begin{bmatrix} \vec{t} \\ 1 \end{bmatrix}_i \\
&= \begin{bmatrix} \cos\theta_i & -\sin\theta_i & l_{i,x} \\ \sin\theta_i & \cos\theta_i & l_{i,y} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \vec{t} \\ 1 \end{bmatrix}_i
\end{aligned}
$$

The complete transformation
for (n+1)-link serial chain robot

$$\begin{bmatrix} \vec{e} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta_0) & -\sin(\theta_0) & 1 \\ \sin(\theta_0) & \cos(\theta_0) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{T}_0} \cdots \underbrace{\begin{bmatrix} \cos(\theta_n) & -\sin(\theta_n) & 1 \\ \sin(\theta_n) & \cos(\theta_n) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{T}_n} \begin{bmatrix} \vec{t} \\ 1 \end{bmatrix}$$

Short hand notation when tool vector is constant

$$
\begin{aligned}
\begin{bmatrix} \vec{e} \\ 1 \end{bmatrix} &= \vec{F}(\vec{\theta}) \\
&= \vec{F}(\vec{\theta}, \vec{t}) \\
&= \mathbf{T}_0 \cdots \mathbf{T}_n \begin{bmatrix} \vec{t} \\ 1 \end{bmatrix}
\end{aligned}
$$

The Inverse Kinematics Problem

$$\begin{bmatrix} \vec{g} \\ 1 \end{bmatrix} = \vec{F}(\vec{\theta})$$

$$\vec{\theta}^* = \arg\min_{\vec{\theta}} \frac{1}{2} \left| \begin{bmatrix} \vec{g} \\ 1 \end{bmatrix} - \vec{F}(\vec{\theta}) \right|$$

When Solving the Optimization Problem

$$\frac{\partial}{\partial \vec{\theta}} \left(\vec{F}(\vec{\theta}) \begin{bmatrix} \vec{t} \\ 1 \end{bmatrix} \right) = ???$$

We often need to compute the gradient wrt parameters