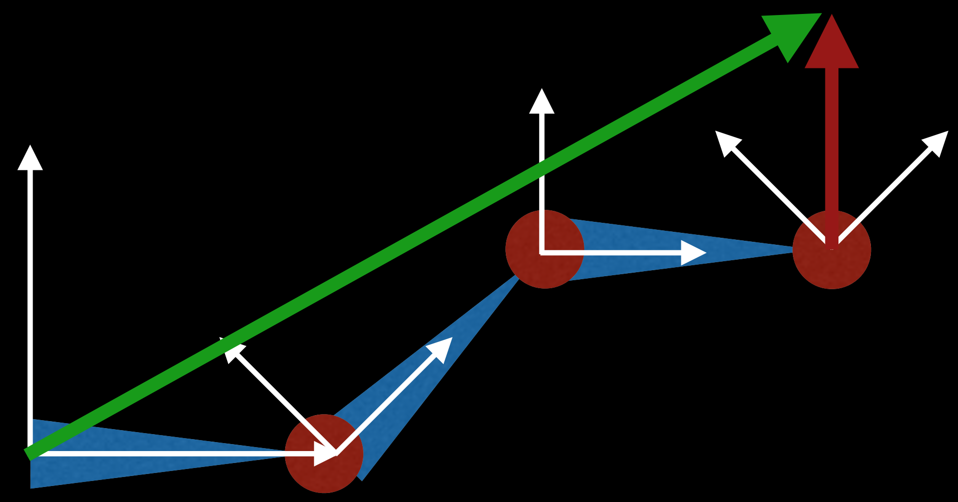
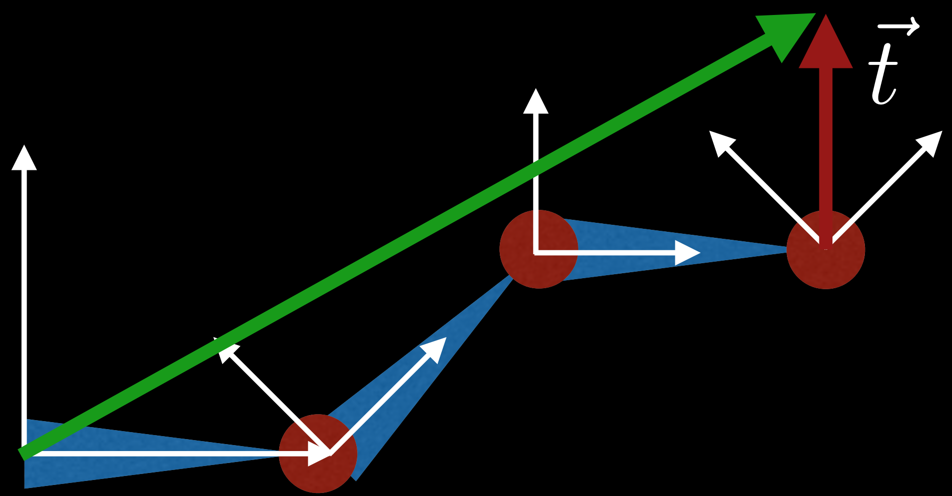
Remember our example

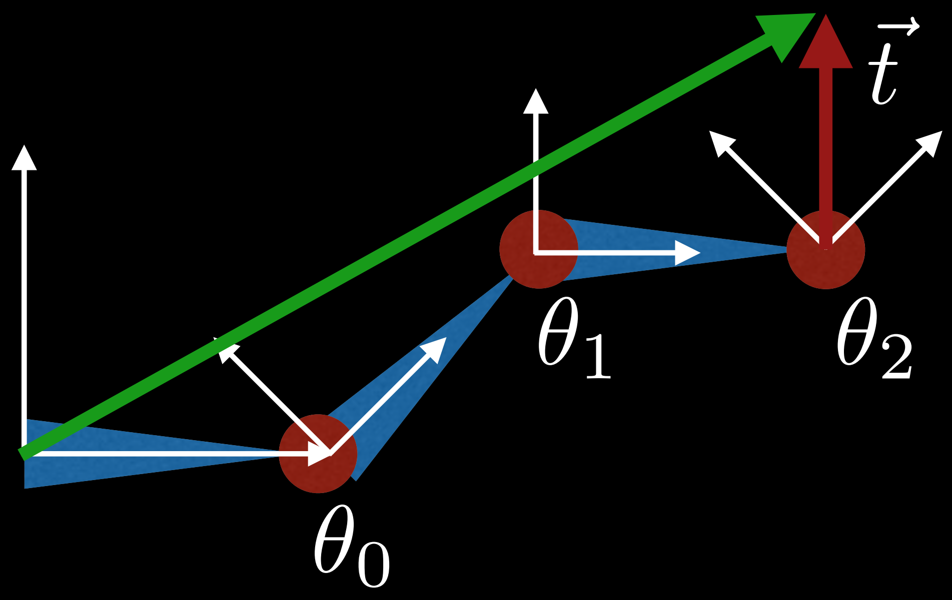


Assume 2D world then we define the tool vector



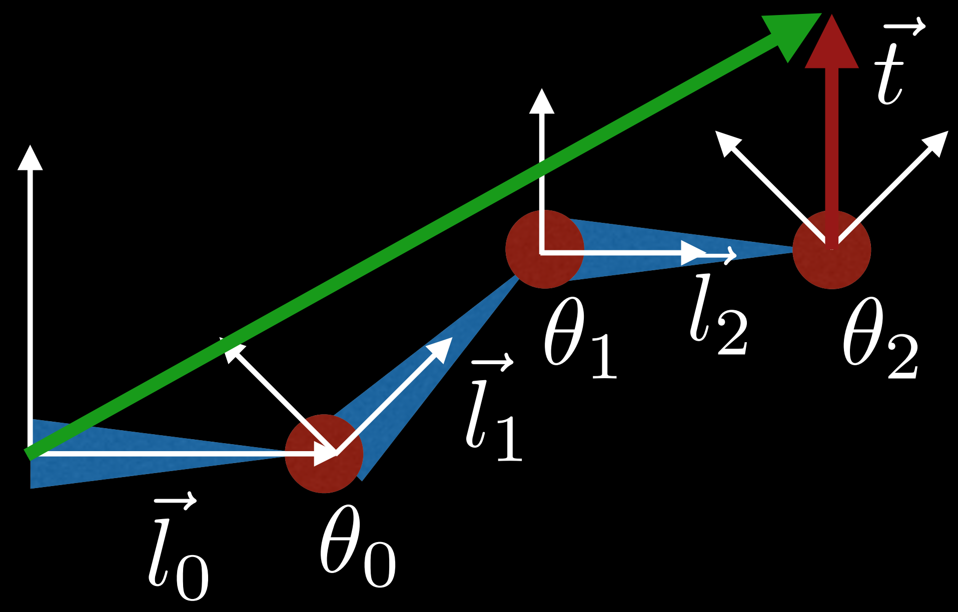
$$\vec{t} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

We define the link angles



$$\vec{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \frac{\pi}{4} \\ -\frac{\pi}{4} \\ \frac{\pi}{4} \end{bmatrix}$$

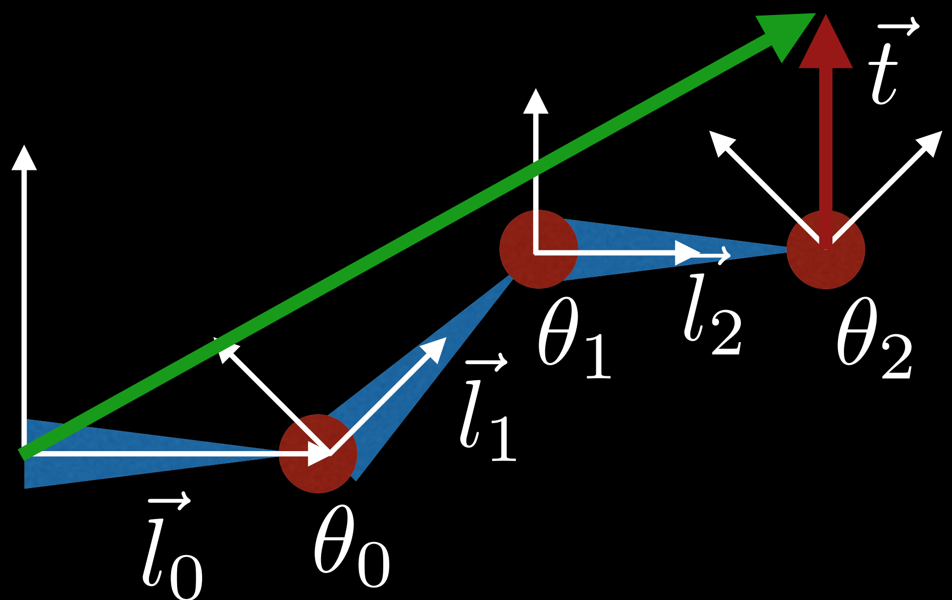
We define the link vectors



$$\vec{l}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

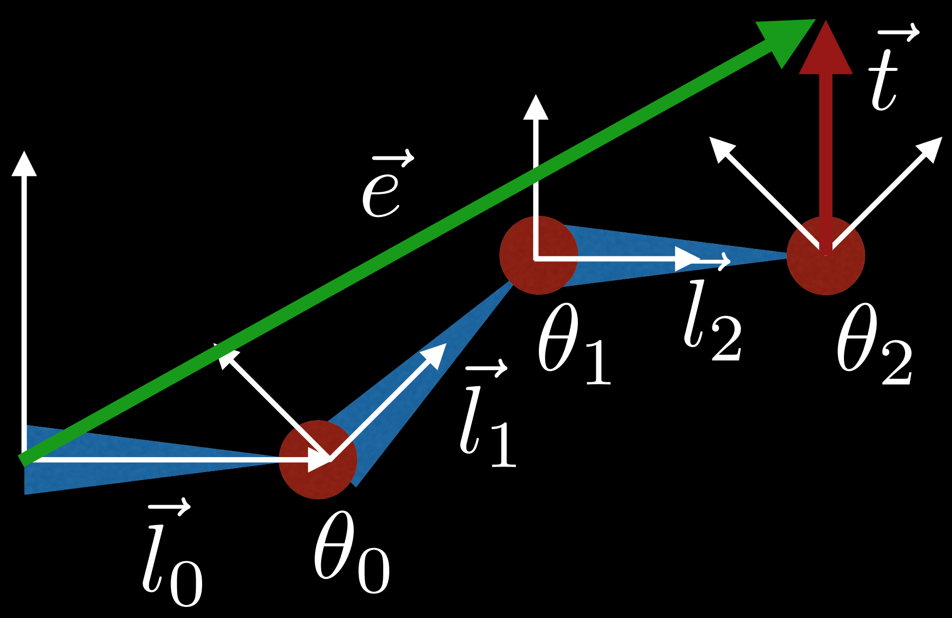
Remember we assume links are rigid…

They could be “free” parameters
instead of constants our example



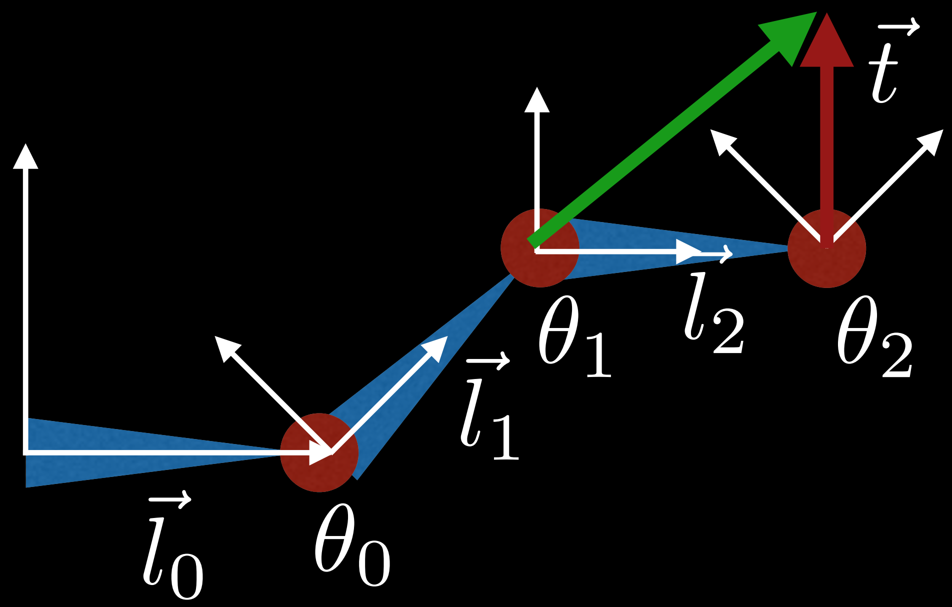
$$\vec{l}_0 = \vec{l}_1 = \vec{l}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The end-effector vector



$$\vec{e} = \begin{bmatrix} e_x \\ e_y \end{bmatrix}$$

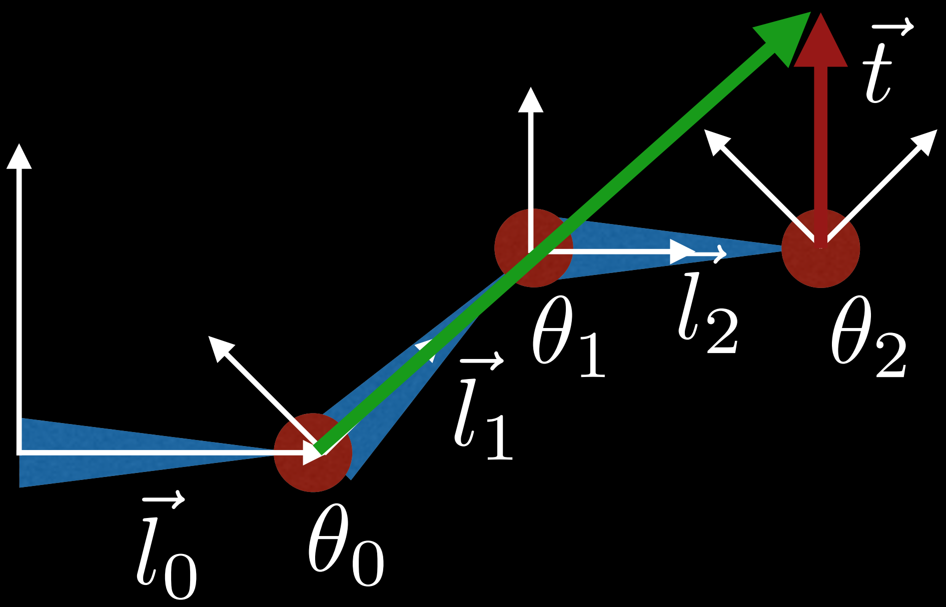
Writing up coordinate transformations



$$\left[\vec{t}\right]_2 = \vec{t}$$

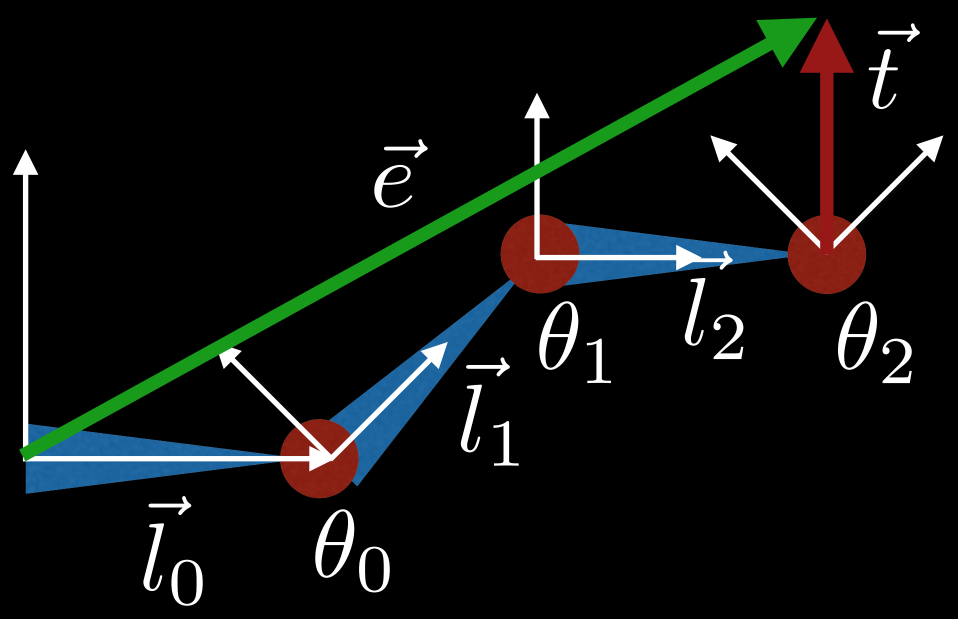
$$\left[\vec{t}\right]_1 = \mathcal{T}(\theta_2, \vec{l_2}) \circ \left[\vec{t}\right]_2 = \mathcal{T}(\theta_2, \vec{l_2}) \circ \vec{t}$$

Writing up coordinate transformations



$$\left[\vec{t}\right]_0 = \mathcal{T}(\theta_1, \vec{l}_1) \circ \left[\vec{t}\right]_1$$
$$= \mathcal{T}(\theta_1, \vec{l}_1) \circ \mathcal{T}(\theta_2, \vec{l}_2) \circ \vec{t}$$

Writing up coordinate transformations



$$\vec{e} = \left[\vec{t}\right]_{\mathrm{wcs}} = \left[\vec{t}\right]_{-1}$$

$$\vec{e} = \mathcal{T}(\theta_0, \vec{l}_0) \circ \left[\vec{t}\right]_0$$

$$= \mathcal{T}(\theta_0, \vec{l}_0) \circ \mathcal{T}(\theta_1, \vec{l}_1) \circ \mathcal{T}(\theta_2, \vec{l}_2) \circ \vec{t}$$

The Final Math Formula

$$
\begin{aligned}
\vec{e} &= \mathcal{T}(\theta_0, \vec{l_0}) \circ \mathcal{T}(\theta_1, \vec{l_1}) \circ \mathcal{T}(\theta_2, \vec{l_2}) \circ \vec{t} \\
&= \mathcal{T}(\theta_0) \circ \mathcal{T}(\theta_1) \circ \mathcal{T}(\theta_2) \circ \vec{t} \\
&= \mathcal{T}_0 \circ \mathcal{T}_1 \circ \mathcal{T}_2 \circ \vec{t} \\
&= \vec{F}(\vec{\theta}) \circ \vec{t} = \vec{F}(\vec{\theta}, \vec{t})
\end{aligned}
$$