Universal Model Inference

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19. april 2001

We have seen the design of Tu and Terzopoulos. This design incorporates the exiting ideas of Reynolds and builds upon Brook's subsumption architecture.

Another design based on information theory is presented in this note. The principle is that the robot builds a model of the environment, in particular of their behavior. The robot infers a model of the objects in its environment. The design is presented using an abstraction of a robot.

We want to infer a model of a stochastic automaton that reads bits and writes bits. For each bit read, it writes a bit. We further specify that we want to infer a Markov source model of a certain order n, but we leave the order n to be determined at each time step. Thus, the model is a varibale order Markov source. For that purpose we invoke the MDL principle.

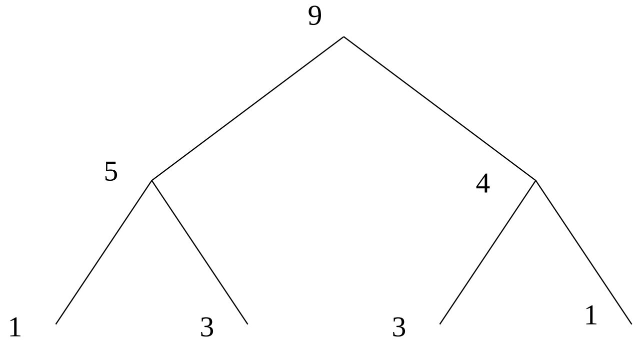
At each time step we determine that Markov source which would have encoded the entire previous bit string as compactly as possible. This determination is made by computing the minimum encoding length of the entire input string up til the just read bit by a Markov source of order n for an interval of orders. This interval can be chosen either fixed as $\{0, \cdots, M\}$ or variable as $\{n-w, \cdots, n+w\}$, where n is the current order.

In order to implement this program we need to find the MDL for a fixed order k. Let N be the number of bits read. To encode the entire string by a order k model, we need the 2^{k-1} frequencies of words of length $k-1$. In addition we need the first k bits to determine the context, that is which distribution to use as the first one. For each number we need $\log_2 N$ bits. Thus, when k is increased by 1, the model cost is increased by $(k + 2^k) - (k - 1 + 2^{k-1}) = 1\text{bit} + 2^{k-1}$ numbers, each of length $\log_2 N$. The incremental model cost is $1 + 2^{k-1} \log_2 N$.

The data encoding cost is determined by the entropy of the model distribution.

Since inference is made over a class of models, it is called universal (over that class of models).

An example will be useful.



Figur 1: String 011010010. The number at a node is the number of occurrences of all substrings of length at most 2.

Example

Figure 1 shows the frequency of subwords of length at most 2 of a string.

Let us consider a Markov model of order 1. Two different distributions are used depending on the immediately preceeding letter. The coding probability distributions are $\{1, 3\}$ after symbol 0 and $\{3, 1\}$ after symbol 1. We have to transmit the first bit in order to determine the first context. The MDL code length becomes $1 + 2^2 \log_2 9 + 4\,\mathrm{H}(\{1, 3\}) + 4\,\mathrm{H}(\{1, 3\}) = 4.2$

The MDL cost when using an order 0 Markov source is $2 \log_2 9 + 9\,\mathrm{H}(\{5, 4\}) = 1.9$

In this example it pays to choose an order 2 Markov source after having read 9 symbols.

Robot Interaction

Suppose a robot wants to control the actions of another robot. A plan is to let it infer the behavior of the other robot as a variable Markov source. It then simulates the action of the other robot by each of its own actions. It chooses that action which induces the most favorable action by the other robot.

It can choose to simulate one step further: For each of its own actions, it predicts the action of the other robot. It simulated furthermore the next action of the other robot for each possible action of its own. It then selects that action which provokes the final most favorable action of the other robot.

This plan can be execute to any depth for which there is computational power available in the allotted time.

Two robots can play a game in which each uses this strategy to predict the other's actions. The robot which uses the most efficient implementation will be the winner.

This strategy is similar to the standard game tree which a game playing program builds, for instance to play chess.