The k-means Algorithm

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March 14, 2001

Given a set of N data $\{x_i\}$, $0 < i \leq N$ from R^n. We want to partition the data into k disjoint classes $\{c_j\}$, $0 < j \leq k$.

The k-means Algorithm

Initially is given a set of data $\{x_i\}$, $0 < i \leq N$ from R^n and a set of class centers $\{\mu_j^0\}$, $0 < j \leq k$ from R^n.

For $t = 1$ until T do {

1. For each datum x_i, and for each class, compute the distance from x_i to μ_j^{t-1}. Let

$$j(i) \quad = \quad \arg\min_j d(x_i, \mu_j^{t-1})$$

   Thus x_i has the shortest distance to $\mu_{j(i)}^{t-1}$.

2. For each datum x_i, assign x_i to class $c_{j(i)}^t(i)$.
3. Compute the new center of each class as the centroid of all items assigned to the class.
   Let $m = \#{c_j}^t$ and let ${c_j}^t = \{\ x_i\ ,\ 0 \leq i \leq m\}$.

$${\mu_j}^t \quad = \quad \sum_{i=1}^{m} \frac{1}{m} x_i$$

}

The result of the algorithm is the final set of centers $\{\mu_j^T\}$, $0 < j \leq k$.